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FACTOR BIAS AND INNOVATIONS: A MICROECONOMIC APPROACH*

by

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*Research for this paper was supported by
The National Science Foundation Grant No. DA 39495

Social Science Working Paper
Number 38
March 1974

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ABSTRACT

This paper contains a microeconomic analysis of the influence of changes in relative prices on the direction of inventive activity. A control-theory model of a firm which produces final output and also performs research and development is developed. It is assumed that the output of R & D is factor-augmenting technical change. An innovation possibility frontier for the firm is defined and conditions are found under which it is convex. The major theorems relate the changes in innovation in response to changing factor prices to the elasticity of substitution in producing final output and to the nature of the production functions for innovation. Two special cases are examined in detail. When current innovation possibilities are appropriately independent of past innovations, the rate of factor augmentation is the same for all factors where relative prices are constant at any level. Comparing time paths with different, constant relative prices gives conditions under which an increase in the price of a factor directs innovation into lines which economize on that factor. A summary of earlier results on similar subjects is included.

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1. This paper is concerned with a classic question in the theory of innovations, namely, the question "does an increase in the wage rate lead towards an increase in the production and use of labor-augmenting innovations?" Section 2 of this paper contains a brief survey of the literature dealing with this question. Most of the existing literature takes a macroeconomic approach to the problem of factor augmentation bias and in addition directs its attention only at the use of innovations, with the menu of innovations available to the society being taken as given exogenously. Our approach in this paper is microeconomic, in order to deal explicitly with the issue of allocation of resources to the production of innovations and with the responsiveness of that production to the price signals provided by the factor markets. Because of aggregation problems the conclusions derived in this paper do not necessarily carry over in a direct fashion to macro models of the economy, but at least certain issues are raised by those conclusions that are relevant to the study of innovation in a macro context.

Briefly, the model that we employ is that of a profit-maximizing firm that produces a final output and also engages in "in house" R & D activities that result in the production of labor- and capital-augmenting innovations specialized to the final product production process. The firm is assumed to be a monopolist in the market for its final product.[1]

We assume that the firm's production activities can be summarized in terms of well-behaved neoclassical production functions. The notion of an "innovations possibilities set" is introduced at the level of the firm, and we show that the set is convex, with strict convexity occurring only if there is decreasing returns to scale in the production of innovations. These results are the microcounterparts of the Kennedy postulate of convexity of the aggregate innovations possibilities set.

The profit-maximizing choices of the firm are derived from a control-theory formulation of the firm's activities. The wage-rental ratio is assumed to be fixed over time, and we attempt to analyze the consequences for labor- and capital-augmenting innovations of a shift in that wage-rental ratio.[2]

By considering a relatively extensive model of the innovating firm, we can examine several complicating features of innovation. The first is the dependence of current innovation possibilities on past innovative activity. The second is the influence of changes in factor prices on the choice of innovative inputs, since the same factors as are augmented may themselves be used to produce innovations. Each of these considerations can produce paradoxical results as to the influence of prices on the direction of technical change. We establish first those few properties of technical change which do not depend on specific assumptions on the form of the dependence of current innovations possibilities on past innovative activity. Then by examining simple cases we show how the complications described make it impossible to state in general how changing factor prices will affect innovation.

Specific results are obtained for two special cases: first, the case in which the percentage rate of increase in augmentation is independent of the levels of such augmentation; and second, the case where the output of factor-augmenting innovations is independent of the levels of augmentation. These two cases exhibit quite different "comparative dynamic" properties and illustrate the dependence of the conclusions reached concerning the

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responsiveness of outputs of innovations, on the assumptions made concerning the characteristics of the production processes employed by the firm.

2. At least since J. R. Hicks' conjecture in the Theory of Wages that increases in the wage rate call forth labor-saving innovations, it has been recognized that changing factor prices may affect innovation. The conjecture is, as we shall show, not obviously true. Nor has it been unchallenged. Fellner (1962) and Salter (1960) have argued precisely the opposite, that although an anticipated change in prices might bias inventive activity, there is no reason to expect a difference in innovation under conditions of continuing high wages than under continuing high profits. The arguments by both authors are based on the idea that the firm does not care what kind of costs are reduced; it simply wants to reduce total costs as quickly as possible.

Two separate points appear at issue. One relates to exogenous trends in innovation or innovation possibilities, and disagreements arise from differing priors on the direction of exogenous trends.[3] The other point, which we address in this paper, is whether and how, in a world with no exogenous trend, innovations will respond to prices.

Despite the important role which technical progress has played in models of economic growth, the problem of determining how the bias in technical change will respond to changes in factor prices has never been the subject of a complete formal analysis. Analysis of induced technical change has concentrated mainly on finding conditions under which the economy will have a long-run balanced growth path consistent with a limited set of "stylized facts." In the major recent treatments of induced innovation [Samuelson (1965), Conlisk (1969), Nordhaus (1969), Drandakis and Phelps (1966)], it is assumed that the quantity of labor to be employed is determined exogenously, and the quantity of capital is determined by

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saving and investment. Equilibrium and optimal growth paths are found by appropriate optimizations using labor and capital supplies to define constraints. This approach is fundamentally macroeconomic.

Although these models focus on the question of how to choose from among an exogenously given set of innovation possibilities, they arrive at results which are special cases of the model in which innovations are literally produced. A brief survey of these results will set the stage for the models of this paper.

Samuelson (1965) examines a formal model of innovation using Kennedy's (1964) idea of an innovation possibility frontier [IPF]. We revise his notation to be consistent with ours. Assume a production function $F(\cdot,\cdot)$ which is homogeneous of degree one. If technical progress is factor augmenting we can define the variables of the production function to be $A(t)K$, $B(t)L$, so that $Y = F(A(t)K, B(t)L)$. The various derivatives will be represented as follows:

$$\frac{\partial F}{\partial[A(t)K]} = F_1 \qquad \frac{\partial F}{\partial[B(t)L]} = F_2$$

$$\frac{dA}{dt} = \dot{A} \qquad \frac{\dot{A}}{A} = a \qquad \frac{dB}{dt} = \dot{B} \qquad \frac{\dot{B}}{B} = b.$$

Some identities will be used frequently:

$$\frac{\partial F}{\partial K} = AF_1 \qquad \frac{\partial F}{\partial L} = BF_2.$$

Since F is homogeneous of degree one we can define

$$v = \frac{A(t)K}{B(t)L} \text{ and } f(v) = F\left(\frac{A(t)K}{B(t)L}, 1\right) = \frac{1}{B(t)L} F(A(t)K, B(t)L)$$

where

$$f'(v) = F_1$$

$$f - vf'(v) = F_2.$$

We can call v the "augmented capital-labor ratio."

The share of capital in output, α_K, can be defined in terms of F_1 or of f'. Let r = price of K, w = price of L. Then

$$\alpha_K = \frac{rK}{Y} = \frac{\partial F}{\partial K}\frac{K}{F} = \frac{AKF_1}{F} = \frac{vf'}{f}$$

Similarly

$$\alpha_L = \frac{wL}{Y} = \frac{\partial F}{\partial L}\frac{L}{F} = \frac{BLF_2}{F} = \frac{f - vf'}{f}$$

$$\frac{\alpha_K}{\alpha_L} = \frac{vf'}{f - vf'}.$$

Following Samuelson we define a cost function

$$C\left(\frac{r}{A(t)}, \frac{w}{B(t)}\right)$$

where $C = \min_{K,L} rK + wL$ subject to

$$F(A(t)K, B(t)L) = 1.$$

Samuelson exploits certain "duality" relations between C and F. Define

$$C_1 = \frac{\partial C}{\partial\left(\frac{r}{A(t)}\right)} \qquad C_2 = \frac{\partial C}{\partial\left(\frac{w}{B(t)}\right)}, \text{ so that } \frac{\partial C}{\partial r} = \frac{C_1}{A(t)} \qquad \frac{\partial C}{\partial w} = \frac{C_2}{B(t)}.$$

Samuelson states that

$$\frac{\partial C}{\partial r} = \frac{K}{F}, \qquad \frac{\partial C}{\partial w} = \frac{L}{F},$$

and that

$$\frac{rK}{CF} = \alpha_K = \frac{K}{F}\frac{\partial F}{\partial K},$$

$$\frac{wL}{CF} = \alpha_L = \frac{L}{F}\frac{\partial F}{\partial L}.$$

Finally, Samuelson assumes an exogenous innovation possibility frontier, written

$$b = f(a)$$

where $f'(a) < 0$, $f''(a) < 0$.

Samuelson assumes that firms act to minimize the instantaneous rate of reduction in unit cost. The rate is obtained by evaluating $\frac{\partial C}{\partial t}/C$.

$$\frac{\partial C}{\partial t} = C_1\frac{d}{dt}\left(\frac{r}{A(t)}\right) + C_2\frac{d}{dt}\left(\frac{w}{B(t)}\right)$$

$$= -\left[C_1 r\frac{\dot{A}}{A^2} + C_2 w\frac{\dot{B}}{B^2}\right] + C_1\frac{\dot{r}}{A} + C_2\frac{\dot{w}}{B}$$

$$= -\left[\frac{\partial C}{\partial r} r\frac{\dot{A}}{A} + \frac{\partial C}{\partial w} w\frac{\dot{B}}{B}\right] + \frac{\partial C}{\partial r}\dot{r} + \frac{\partial C}{\partial w}\dot{w}$$

$$= \left[\frac{rK}{F}\cdot\frac{\dot{A}}{A} + \frac{wL}{F}\cdot\frac{\dot{B}}{B}\right] + \frac{K}{F}\dot{r} + \frac{L}{F}\dot{w}.$$

$$-\frac{\dot{C}}{C} = \frac{rK}{CF}a + \frac{wL}{CF}b - \left[\frac{K}{CF}\dot{r} + \frac{L}{CF}\dot{w}\right]$$

$$= \alpha_K a + \alpha_L b - \left[\frac{K\dot{r} + L\dot{w}}{CF}\right]$$

The firm chooses a and b to maximize this expression subject to $b = f(a)$. Substituting and differentiating we have

$$\frac{d}{da}\left[\alpha_K a + \alpha_L f(a)\right]$$

$$= \alpha_K + \alpha_L f'(a) = 0$$

$$f'(a) = -\frac{\alpha_K}{\alpha_L}.$$

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Under the assumed conditions $f'(a) < 0$, $f''(a) < 0$ we can solve for the inverse function

$$g\left(\frac{\alpha_K}{\alpha_L}\right) = f'^{[-1]}\left(\frac{\alpha_K}{\alpha_L}\right),$$

where $g' = \frac{1}{f''} < 0$. By successive eliminations we can obtain an expression

$$-a = h\left(\frac{r}{A} / \frac{w}{B}\right).$$

By using the first order conditions we obtain α_K and α_L in terms of AK and BL; further substitution enables us to express AK and BL in terms of the ratio $\frac{r}{A} / \frac{w}{B}$. Thus

$$h' = g' \cdot \frac{d\left(\frac{\alpha_K}{\alpha_L}\right)}{d\left(\frac{AK}{BL}\right)} \cdot \frac{d\left(\frac{AK}{BL}\right)}{d\left(\frac{r}{A} / \frac{w}{B}\right)}$$

We can determine the sign of each derivative. By assumption

$$\operatorname{sgn} g' = -.$$

By convexity

$$\operatorname{sgn} \frac{d\left(\frac{AK}{BL}\right)}{d\left(\frac{r}{A} / \frac{w}{B}\right)} = -.$$

The sign of $d\left(\frac{\alpha_K}{\alpha_L}\right) / d\left(\frac{AK}{BL}\right)$ depends on the elasticity of substitution.

$$\frac{d\left(\frac{\alpha_1}{\alpha_2}\right)}{d\left(\frac{AK}{BL}\right)} = \frac{d}{dv}\left(\frac{vf'}{f - vf'}\right)$$

$$= \frac{(f - vf')(vf'' + f') - vf'(f' - vf'' - f')}{(f - vf')^2}$$

$$= \frac{vff'' + f'(f - vf')}{(f - vf')^2}$$

$$= \frac{f'vff''}{f'(f - vf')^2} + \frac{f'(f - vf')}{(f - vf')^2}.$$

Since $\sigma = -\frac{f'(f - vf')}{vff''}$,

$$\frac{d\left(\frac{\alpha_K}{\alpha_L}\right)}{d\left(\frac{AK}{BL}\right)} = \frac{f'}{-\sigma} + \frac{f'}{f - vf'}$$

$$= \frac{f' - \sigma f'}{-\sigma(f - vf')}$$

$$= \frac{\sigma - 1}{\sigma} \frac{f'}{f - vf'}.$$

Since $f' > 0$ and $f - vf' > 0$ by assumption,

$$\frac{d\left(\frac{\alpha_K}{\alpha_L}\right)}{d\left(\frac{AK}{BL}\right)} \gtrless 0 \longleftrightarrow \sigma \gtrless 1.$$

Therefore $h' \gtrless 0 \longleftrightarrow \sigma \gtrless 1$.

Drandakis and Phelps (1966) assume that the firm maximizes the instantaneous proportionate rate of growth in output with fixed inputs. Writing $\dot{Y} = F(K, L, t)$, they define

$$R = \frac{F_t}{F},$$

which is to be maximized. Bias they define as

$$D = \frac{\partial^2 F}{\partial K \partial t} - \frac{\partial^2 F}{\partial L \partial t} = M_K - M_L.$$

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Assuming factor augmentation, they obtain an expression

$$R = \frac{\partial F[A(t)K, B(t)L]}{\partial t} \Big/ F$$

$$= \frac{F_1 K\dot{A}}{F} + \frac{F_2 L\dot{B}}{F} = \frac{\dot{A}}{A}\frac{\frac{\partial F}{\partial K}K}{F} + \frac{\dot{B}}{B}\frac{\frac{\partial F}{\partial L}L}{F}$$

$$= \alpha_K \frac{\dot{A}}{A} + \alpha_L \frac{\dot{B}}{B}$$

$$= \alpha_K a + \alpha_L b,$$

which is the same maximand as Samuelson's.

In an Appendix, Drandakis and Phelps derive the following expression for bias

$$D = \frac{1-\sigma}{\sigma}(b - a).$$

Thus whether or not an increase in the augmentation coefficient for a factor results in a reduction in the use of that factor (in natural units) depends on the elasticity of substitution. For $\sigma < 1$ the expected results hold.

3. We next examine the properties of a model of in house R & D conducted by a firm that produces a final product that incorporates the innovations produced by the R & D labs. We are particularly interested in the way in which profit-maximizing behavior leads to an allocation of resources between the production of capital-augmenting and labor-augmenting innovations in response to the market wage-rental situation.

The model is as simple as possible. The firm is assumed to be a monopolist in the market for its final output Y, produced using labor L_1 and capital K_1 hired in competitive factor markets. Labor and capital used in the production of the final product are augmented by innovations produced in the R & D operations of the firm. Thus AK_1 and BL_1 represent the effective amounts of capital and labor employed in producing the final output.

R & D activities are summarized in two production activities, one producing capital-augmenting innovations, $\dot{A}$, with the other producing labor-augmenting innovations, $\dot{B}$. Innovations are assumed to be specialized to labor and capital employed in producing the firm's final output, so that the labor and capital used in the R & D departments are not augmented by the innovations produced. L_2 and K_2 represent labor and capital employed to produce $\dot{A}$ while L_3 and K_3 denote labor and capital used in the production of $\dot{B}$.

Before turning to the conditions characterizing a profit-maximizing time path for the firm, we first explore the properties of the "innovations possibilities set" for the firm. The "in-house" innovations possibilities set is defined as those combinations of $\dot{A}$ and $\dot{B}$ that can be obtained for a given cost in terms of capital and labor, assuming that A and B are fixed.

To obtain the outer boundary of this set, we solve the following problem.

$$\text{Max } \dot{A} = \phi(K_2, L_2, A, B)$$

subject to (1) $\dot{B} = \overline{C}$

(where $\dot{B} = \Psi(K_3, L_3, A, B)$);

(2) $w(L_2 + L_3) + r(K_2 + K_3) = \overline{M}$

ϕ and Ψ are assumed to be homogeneous of some positive degrees less than or equal to 1, and ϕ and Ψ are quasi-concave.

Let $H = \phi + \lambda_1(\Psi - \bar{C}) + \lambda_2(\bar{M} - w(L_2 + L_3) - r(K_2 + K_3))$.

At a constrained maximum we have

(i) $\phi_K - r\lambda_2 = 0$

(ii) $\phi_L - w\lambda_2 = 0$

(iii) $\lambda_1\Psi_K - r\lambda_2 = 0$

(iv) $\lambda_1\Psi_L - w\lambda_2 = 0$

(v) $\Psi - \bar{C} = 0$

(vi) $\bar{M} - w(L_2 + L_3) - r(K_2 + K_3) = 0$

Let

$$A^* = \begin{bmatrix} \phi_{KK} & \phi_{KL} & 0 & 0 & 0 & -r \\ \phi_{LK} & \phi_{LL} & 0 & 0 & 0 & -w \\ 0 & 0 & \Psi_{KK} & \Psi_{KL} & \Psi_K & -r \\ 0 & 0 & \Psi_{LK} & \Psi_{LL} & \Psi_L & -w \\ 0 & 0 & \Psi_K & \Psi_L & 0 & 0 \\ -r & -w & -r & -w & 0 & 0 \end{bmatrix}$$

then at a regular constrained maximum, A^* has the property that $|A^*| > 0$.

The slope of the boundary of the innovations possibilities set is given by

$$\frac{d\dot{A}}{d\dot{B}} = \frac{\phi_K dK_2 + \phi_L dL_2}{\Psi_K dK_3 + \Psi_L dL_3}$$

From (2), $w dL_2 + r dK_2 = -(w dL_3 + r dK_3)$

while from (i) - (iv),

$$\frac{d\dot{A}}{d\dot{B}} = \frac{r\lambda_2 dK_2 + w\lambda_2 dL_2}{r\left(\frac{\lambda_2}{\lambda_1}\right)dK_3 + w\left(\frac{\lambda_2}{\lambda_1}\right)dL_3} = -\lambda_1$$

It thus follows that the innovations possibilities set is convex if $\frac{d\lambda_1}{dB} \geq 0$.

Differentiating the first order conditions with respect to $\dot{B}$ we obtain the system

$$\begin{bmatrix} \phi_{KK} & \phi_{KL} & 0 & 0 & 0 & -r \\ \phi_{LK} & \phi_{LL} & 0 & 0 & 0 & -w \\ 0 & 0 & \Psi_{KK} & \Psi_{KL} & \Psi_K & -r \\ 0 & 0 & \Psi_{LK} & \Psi_{LL} & \Psi_L & -w \\ 0 & 0 & \Psi_K & \Psi_L & 0 & 0 \\ -r & -w & -r & -w & 0 & 0 \end{bmatrix} \begin{bmatrix} dK_2 \\ dL_2 \\ dK_3 \\ dL_3 \\ d\lambda_1 \\ d\lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ d\dot{B} \\ 0 \end{bmatrix}$$

Hence $\frac{d\lambda_1}{d\dot{B}} = \frac{A^*_{55}}{|A^*|}$ where A^*_{55} is the cofactor formed by deleting the fifth row and column from A^*.

By block multiplication we have

$$A^*_{55} = \begin{vmatrix} \phi_{KK} & \phi_{KL} \\ \phi_{LK} & \phi_{LL} \end{vmatrix} \cdot \begin{vmatrix} \Psi_{KK} & \Psi_{KL} & -r \\ \Psi_{LK} & \Psi_{LL} & -w \\ -r & -w & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \Psi_{KK} & \Psi_{KL} \\ \Psi_{LK} & \Psi_{LL} \end{vmatrix} \cdot \begin{vmatrix} \phi_{KK} & \phi_{KL} & -r \\ \phi_{LK} & \phi_{LL} & -w \\ -r & -w & 0 \end{vmatrix}$$

If ϕ and Ψ are homogeneous of degree one, then $A^*_{55} = 0$ and $\frac{d\lambda_1}{d\dot{B}} = 0$. If ϕ and Ψ are homogeneous of positive degree less than one,

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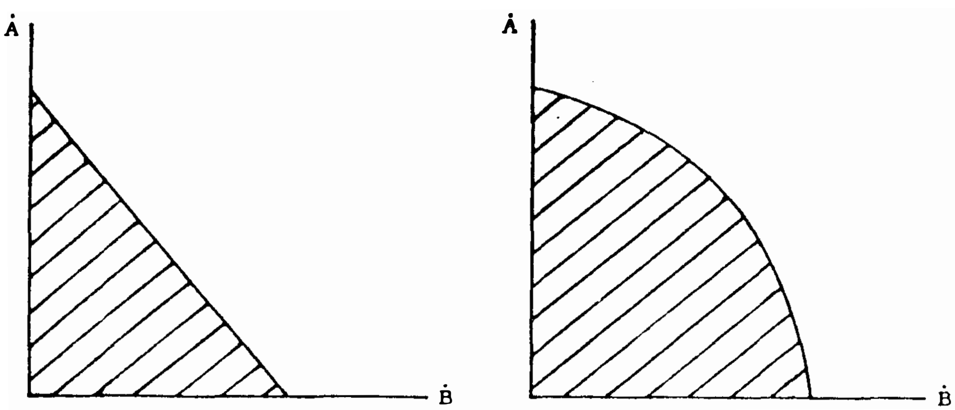
note that $\begin{vmatrix} \Psi_{KK} & \Psi_{KL} & -r \\ \Psi_{LK} & \Psi_{LL} & -w \\ -r & -w & 0 \end{vmatrix} = \left(\frac{\lambda_1}{\lambda_2}\right)^2 \begin{vmatrix} \Psi_{KK} & \Psi_{KL} & \Psi_K \\ \Psi_{LK} & \Psi_{LL} & \Psi_L \\ \Psi_K & \Psi_L & 0 \end{vmatrix}$

while $\begin{vmatrix} \phi_{KK} & \phi_{KL} & -r \\ \phi_{LK} & \phi_{LL} & -w \\ -r & -w & 0 \end{vmatrix} = \left(\frac{1}{\lambda_2}\right)^2 \begin{vmatrix} \phi_{KK} & \phi_{KL} & \phi_K \\ \phi_{LK} & \phi_{LL} & \phi_L \\ \phi_K & \phi_L & 0 \end{vmatrix}$

Under quasi-concavity and positive homogeneity of degree less than one, both of these expressions are positive.

Hence we conclude that when ϕ and Ψ exhibit constant returns, the IP set is convex with a straight line outer boundary. Under decreasing returns to scale, homogeneity and quasi-concavity, the IP set is convex with the outer boundary a strictly concave function. The cases are shown below.



Constant returns to ϕ and Ψ — Decreasing returns to ϕ and Ψ

The In-House I. P. Set

(Note that because A and B are assumed fixed, precisely the same diagrams appear if we replace $\dot{A}$ by $\dot{A}/A$ and $\dot{B}$ by $\dot{B}/B$).

Turning to the implications of profit maximization for the allocation of resources within the firm, the firm's problem may be formulated as follows.

$$\max \int_0^\infty [p(Y)F(AK_1, BL_1) - wL - rK]e^{-\delta t}dt$$

Subject to $\dot{A} = \phi(A, B, K_2 L_2)$

$\dot{B} = \Psi(A, B, K_3 L_3)$

$A(0) = A_0 \qquad B(0) = B_0$

where $L = L_1 + L_2 + L_3$

$K = K_1 + K_2 + K_3, \qquad Y = F(AK_1, BL_1)$

Let $H = [pF - wL - rK]e^{-\delta t} + \lambda_1\phi + \lambda_2\Psi$

First order conditions are given by:[4]

(1) $[(MR)F_1 A - r]e^{-\delta t} = 0$

(2) $[(MR)F_2 B - w]e^{-\delta t} = 0$

(3) $-re^{-\delta t} + \lambda_1\phi_K = 0$

(4) $-we^{-\delta t} + \lambda_1\phi_L = 0$

(5) $-re^{-\delta t} + \lambda_2\Psi_K = 0$

(6) $-we^{-\delta t} + \lambda_2\Psi_L = 0$

(7) $\dot{\lambda}_1 = -(MR)F_1 K_1 e^{-\delta t} - \lambda_1\phi_A - \lambda_2\Psi_A$

(8) $\dot{\lambda}_2 = -(MR)F_2 L_1 e^{-\delta t} - \lambda_1\phi_B - \lambda_2\Psi_B$

(9) $\dot{A} = \phi$

(10) $\dot{B} = \Psi$

with transversality conditions $\lim_{t\to\infty} \lambda_1 = 0$, $\lim_{t\to\infty} \lambda_2 = 0$;

where $MR = p + \frac{dp}{dY}$, $F_1 = \frac{\partial F}{\partial(AK_1)}$, $F_2 = \frac{\partial F}{\partial(BL_1)}$. [5]

We will work with the special case in which $p(Y)$ is assumed to satisfy $\lim_{Y \to 0} p(Y) = +\infty$, $\lim_{Y \to +\infty} p(Y) = 0$, $p(Y) \geq 0$ for all $Y \geq 0$ with $\frac{dp}{dY} < 0$ for all $Y \geq 0$. Further, F, ϕ, and Ψ are assumed to be "well behaved" neoclassical production functions. In particular F is homogeneous of degree one in AK_1, BL_1, while ϕ and Ψ are homogeneous of degree one in K_2, L_2, and K_3, L_3 respectively.

Thus $F(AK_1, BL_1) = BL_1 F(v_1, 1) \equiv BL_1 f(v_1)$ where $v_1 = \frac{AK_1}{BL_1}$. Further, $\lim_{v_1 \to 0} f(v_1) = +\infty$, $\lim_{v_1 \to +\infty} f(v_1) = 0$, $f(v_1) \geq 0$ for $v_1 \geq 0$ and $f'(v_1) > 0$, $f''(v_1) < 0$ for $v_1 \geq 0$.

Similarly, let $\phi(A, B, K_2, L_2) = \alpha(A)G(K_2, L_2) = \alpha(A)L_2 g(v_2)$ while $\Psi(A, B, K_3, L_3) = \gamma(B)H(K_3, L_3) = \gamma(B)L_3 h(v_3)$ where $v_2 = \frac{K_2}{L_2}$, $v_3 = \frac{K_3}{L_3}$. g and h are assumed to obey the same neoclassical properties as f.[6]

Finally, let $\mu_1 = \lambda_1 e^{\delta t}$, $\mu_2 = \lambda_2 e^{\delta t}$. Thus the first order conditions may be rewritten as follows.

(1') $[(MR)Af' - r]e^{-\delta t} = 0$

(2') $[(MR)B(f - v_1 f') - w]e^{-\delta t} = 0$

(3') $-r + \mu_1 g'\alpha = 0$

(4') $-w + \mu_1(g - v_2 g')\alpha = 0$

(5') $-r + \mu_2 h'\gamma = 0$

(6') $-w + \mu_2(h - v_3 h')\gamma = 0$

(7') $\dot{\mu}_1 = \delta\mu_1 - (MR)f'K_1 - \mu_1\alpha' L_2 g$

(8') $\dot{\mu}_2 = \delta\mu_2 - (MR)(f - v_1 f')L_1 - \mu_2\gamma' L_3 h$

(9') $\dot{A} = \alpha(A)L_2 g(v_2)$

(10') $\dot{B} = \gamma(B)L_3 h(v_3)$

Let $\omega = \frac{w}{r}$. Then (1') - (6') can be used to establish that

$$\omega = \frac{B}{A}\left[\frac{f - v_1 f'}{f'}\right] = \frac{g - v_2 g'}{g'} = \frac{h - v_3 h'}{h'}$$

We are particularly concerned with the case where ω is constant over time. For that case we have

$$\dot{\omega} = 0 = \frac{-gg''}{(g')^2}\dot{v}_2 = \frac{-hh''}{(h')^2}\dot{v}_3$$

Under the neoclassical conditions, v_2 and v_3 are uniquely determined by ω, and are constant over time when ω is constant.

Further, given ω, A and B, v_1 is uniquely determined, while $\dot{\omega} = 0$ implies that

$$\frac{\dot{v}_1}{v_1} = \sigma_f \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)$$

where σ_f is the elasticity of substitution between augmented capital and augmented labor in the production of the firm's final product.

Let $q_1 = \frac{K_1}{L_1}$ so that $v_1 = \frac{A}{B}q_1$. We then obtain

$$(*) \quad \frac{\dot{q}_1}{q_1} = (\sigma_f - 1)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right).$$

This result corresponds to that obtained by Samuelson (1965). It asserts the following.

(i) If $\sigma_f = 1$ so that F is Cobb-Douglas, the capital-labor ratio in producing the firm's final product is independent of the relative rates of innovation so far as capital and labor augmentation is concerned;

(ii) If $\sigma_f > 1$, then the capital-labor ratio in producing the final product increases if the rate of increase in capital-augmenting innovations exceeds the rate of increase in labor-augmenting innovations, and conversely;

(iii) If $\sigma_f < 1$, the capital-labor ratio in producing the final product falls if the rate of increase in capital-augmenting innovations exceeds the ratio of increase in labor-augmenting innovations, and conversely.

Thus far we have only exploited the cost-minimizing properties of the model. The properties that follow from profit maximization can be derived as follows.

Note that conditions (1') - (10') are all identities in t. Differentiating (3') with respect to t gives

$$-\dot{r} + \mu_1 g'\alpha'\dot{A} + \mu_1 \alpha g''\dot{v}_2 + \alpha g'\dot{\mu}_1 = 0$$

For $\dot{\omega} = 0$, $\dot{v}_2 = 0$ and $\dot{\mu}_1 = \frac{\dot{r}}{\alpha g'} - \mu_1 \frac{\alpha'\dot{A}}{\alpha}$.

From (7'), $\dot{\mu}_1 = \delta\mu_1 - (MR)f'K_1 - \mu_1\alpha' L_2 g$.

Since $\dot{A} = \alpha L_2 g$, we have

$$(MR)f'K_1 = \delta\mu_1 + \frac{\dot{r}}{\alpha g'} = \frac{\delta r + \dot{r}}{\alpha g'}.$$

Similarly, using (5') and (8') we have

$$(MR)(f - v_1 f')L_1 = \frac{\delta r + \dot{r}}{\gamma h'}.$$

It follows that

$$\left(\frac{f'}{f - v_1 f'}\right)q_1 = \frac{\gamma h'}{\alpha g'},$$

which implies $q_1 = \frac{A}{B}\omega\left(\frac{\gamma h'}{\alpha g'}\right)$, in turn leading to

$$(**) \quad \frac{\dot{q}_1}{q_1} = \left(\frac{\dot{A}}{B} - \frac{\dot{B}}{B}\right) + \frac{\gamma'\dot{B}}{\gamma} - \frac{\alpha'\dot{A}}{\alpha}.$$

This condition might be contrasted with that derived earlier, namely

$$(*) \quad \frac{\dot{q}_1}{q_1} = (\sigma_f - 1)\left(\frac{\dot{A}}{A} - \frac{\dot{A}}{B}\right)$$

Both (*) and (**) must hold simultaneously along a profit-maximizing time path. There are of course a multitude of cases that might be of interest in the study of factor bias and innovation. Here we look only at those cases in which a "quasi-steady state" is achieved in the sense that the capital-labor ratios in all three activities (F, ϕ, Ψ) are constant over time, i.e., cases in which $\dot{q}_1 = 0$ is an identity in t.[7]

For such cases, the analysis of "comparative dynamics" is particularly simple, especially in attempting to determine the impact on the system of a change in the wage-rentel ratio ω.

Case 1. $\dot{A}/A$ and $\dot{B}/B$ independent of A and B.

Recently, Ahmad (1966) and Nordhaus (1973) have emphasized the critical importance of the level of technological progress on the rate of advance of such progress.[8] Their comments are best understood by considering the case in which the rate of progress is independent of the level already achieved. In terms of the model developed here, this is the case where $\alpha(A) = C_1 A$, $\gamma(B) = C_2 B$ for some constants C_1, C_2. We take C_1, C_2 to be unity. From (**) $\dot{q}_1 = 0$ for all t with $q_1 = \omega\left(\frac{h'}{g'}\right)$. If $\sigma_f \neq 1$, then $\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0$ from (*) and hence $v_1 = \frac{A}{B}q_1$ is also constant over time at the level $v_1 = \frac{A}{B}\omega\left(\frac{h'}{g'}\right)$. Since $\frac{\dot{A}}{A} = L_2 g(v_2)$, $\frac{\dot{B}}{B} = L_3 h(v_3)$, we have $L_2 g(v_2) = L_3 h(v_3)$.

Consider a change in ω and its effects on the system.

From $q_1 = \omega\frac{h'}{g'}$, we have

$$\frac{dq_1}{d\omega} = \omega\left(\frac{g'h''\frac{dv_3}{d\omega} - h'g''\frac{dv_2}{d\omega}}{(g')^2}\right) + \frac{h'}{g'}$$

But $\quad \frac{dv_3}{d\omega} = \frac{-(h')^2}{hh''} > 0, \; \frac{dv_2}{d\omega} = \frac{-(g')^2}{gg''} > 0,$

hence $\quad \left(\frac{dq_1}{d\omega} = \omega \frac{-(h')^2}{g'h} + \frac{h'}{g}\right) + \frac{h'}{g'}$

$$= \omega \frac{h'}{g'}\left(\frac{-h'}{h} + \frac{g'}{g} + \frac{1}{\omega}\right)$$

$$\therefore \quad \frac{1}{q_1}\frac{dq_1}{d\omega} = \left[\frac{-1}{\omega + v_3} + \frac{1}{\omega + v_2} + \frac{1}{\omega}\right] > 0.$$

<u>Thus, when $\dot{A}/A$ and $\dot{B}/B$ are independent of A and B, an increase in the wage-rental ratio increases the capital-labor ratios in both final goods production and in the R & D operations of the firm.</u>

Further, $\omega = \frac{B}{A}\left(\frac{f - v_1 f'}{f'}\right)$ so that

$$d\omega = \left(\frac{f - v_1 f'}{f'}\right) d(B/A) - B/A\left(\frac{ff''}{(f')^2}\right) dv_1$$

and $\quad \frac{d\omega}{\omega} = \frac{A}{B} d(B/A) + \frac{1}{\sigma_f}\frac{dv_1}{v_1},$

where $\quad dv_1 = \frac{A}{B} dq_1 + q_1 d(A/B)$

with $\quad \frac{dv_1}{v_1} = \frac{dq_1}{q_1} + \frac{B}{A} d(A/B)$

$$= \frac{dq_1}{q_1} - \frac{A}{B} d(B/A).$$

$$\therefore \quad \frac{d\omega}{\omega} = \frac{A}{B} d(B/A)\left[\frac{\sigma_f - 1}{\sigma_f}\right] + \frac{1}{\sigma_f}\frac{dq_1}{q_1}.$$

For $\sigma_f \neq 1$,

$$\frac{A}{B}\frac{d(B/A)}{d\omega} = \left[\frac{\sigma_f}{\sigma_f - 1}\right]\left\{\frac{1}{\omega} - \frac{1}{\sigma_f q_1}\frac{dq_1}{d\omega}\right\}$$

$$= \left[\frac{\sigma_f}{\sigma_f - 1}\right]\left\{\frac{1}{\omega} - \frac{1}{\sigma_f}\left(\frac{1}{\omega} + \frac{1}{\omega + v_2} - \frac{1}{\omega + v_3}\right)\right\}$$

and $\quad \frac{\omega}{(B/A)}\frac{d(B/A)}{d\omega} = \left[\frac{\sigma_f}{\sigma_f - 1}\right]\left\{1 - \frac{1}{\sigma_f}\left(1 + \frac{\omega}{\omega + v_2} - \frac{\omega}{\omega + v_3}\right)\right\}$

$$= \frac{1}{\sigma_f - 1}\left\{\left(\sigma_f - 1\right) + \frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)}\right\}$$

$$= 1 + \frac{1}{(\sigma_f - 1)}\frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)}$$

Thus the sign of $\frac{d(B/A)}{d\omega}$ depends on the elasticity of substitution of augmented capital and labor in producing the final output, together with the relative capital intensities of labor and capital-augmenting innovations. In general we have

$$\frac{d(B/A)}{d\omega} \gtrless 0 \iff \frac{1}{\sigma_f - 1}\frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)} \gtrless -1$$

Still, it is worthwhile identifying explicitly certain cases in which the "expected" result occurs in that an increase in the wage-rental ratio increases the output of labor-augmenting innovations relative to capital-augmenting innovations.

<u>First, if both innovative activities have the same capital intensities, then B/A increases with increases in ω.</u>

<u>Second, $\sigma_f > 1$ and $v_3 > v_2$; that is, if labor and capital are good substitutes for one another so far as the final good is concerned, and labor-augmenting innovations are relatively more capital intensive than capital-augmenting innovations, B/A increases with ω.</u>

Third, $\sigma_f \le 1$ and $v_2 > v_3$, the case in which there is poor substitutability in producing the final good and capital-augmenting innovations are more capital intensive than labor-augmenting innovations, B/A again increases with ω.

Thus even in the special case where rates of change of innovative activity are independent of the levels achieved, the question as to whether an increase in the wage-rental ratio leads to concentration on labor-augmenting innovations rests on the empirical properties of the production relations within the firm.

Case 2. $\dot{A}$ and $\dot{B}$ independent of A and B.

A second case in which quasi-steady states occur is that in which the outputs of the innovative activities (and not simply the percentage rates of change of such outputs) are independent of the levels achieved. In terms of our model, this is the case where $\alpha(A) = C_1$, $\gamma(B) = C_2$ for some fixed constants C_1 and C_2. To simplify things, we take these constants both to be unity. By (**) we have

$$\frac{\dot{q}_1}{q_1} = \frac{\dot{A}}{A} - \frac{\dot{B}}{B} + \frac{\gamma'\dot{B}}{\gamma} - \frac{\alpha'\dot{A}}{\alpha} = \frac{\dot{A}}{A} - \frac{\dot{B}}{B},$$

on the other hand, by (*)

$$\frac{\dot{q}_1}{q_1} = (\sigma_f - 1)\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)$$

Hence if $\sigma_f \neq 2$, A/B is constant over time, with $q_1 = \frac{A}{B}\omega\left(\frac{h'}{g'}\right)$

Then $$dq_1 = \frac{A}{B}\left(\frac{h'}{g'}\right)d\omega + \frac{h'}{g'}\omega\, d(A/B) + \frac{A}{B}\omega\frac{\left(g'h''dv_3 - h'g''dv_2\right)}{(g')^2}$$

with $$\frac{1}{q_1}\frac{dq_1}{d\omega} = \frac{1}{\omega} + B/A\,\frac{d(A/B)}{d\omega} + \left(\frac{-1}{\omega + v_3} + \frac{1}{\omega + v_2}\right)$$

From $$\omega = \frac{B}{A}\left(\frac{f - v_1 f'}{f'}\right)$$

$$\left(\frac{1 - \sigma_f}{\sigma_f}\right)\frac{A}{B}\frac{d(B/A)}{d\omega} = \left\{\frac{1}{\omega} - \frac{1}{\sigma_f q_1}\frac{dq_1}{d\omega}\right\}$$

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Substituting for $\frac{1}{q_1}\frac{dq_1}{d\omega}$, we obtain

$$\frac{A}{B}\frac{d(B/A)}{d\omega} = -\left\{\frac{1}{\omega}\left(1 - \frac{1}{\sigma_f}\right) - \frac{1}{\sigma_f}\left(\frac{1}{\omega + v_2} - \frac{1}{\omega + v_3}\right)\right\}$$

$$\omega\frac{A}{B}\frac{d(B/A)}{d\omega} = \frac{1}{\sigma_f}\left\{(1 - \sigma_f) - \left(\frac{\omega}{\omega + v_2} - \frac{\omega}{\omega + v_3}\right)\right\}$$

$$= \frac{1}{\sigma_f}\left\{1 - \sigma_f - \frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)}\right\}$$

Thus for $\sigma_f < 1$ and $v_3 \ge v_2$, an increase in the wage-rental ratio leads to an expansion of output of labor-augmenting innovations relative to capital-augmenting innovations, while if $\sigma_f > 1$ and $v_2 \ge v_3$ the opposite result obtains. Note that these are quite different cases than those identified in Case 1 above.

Finally, consider the case where $\alpha(A) = A^m$, $\gamma(B) = B^m$.

By (**), $$\frac{\dot{q}_1}{q_1} = (1 - m)\left[\frac{\dot{A}}{B} - \frac{\dot{B}}{B}\right].$$

For $\sigma_f \neq 2 - m$, (*) implies $\frac{\dot{q}_1}{q_1} = 0$ with A/B constant over time,

and with $q_1 = \omega\left(\frac{h'}{g'}\right)\left(\frac{B}{A}\right)^{m-1}$

Thus $$dq_1 = \left(\frac{B}{A}\right)^{m-1}\left(\frac{h'}{g'}\right)d\omega + \frac{h'}{g'}\omega(m - 1)\left(\frac{B}{A}\right)^{m-2}d(B/A) + \left(\frac{B}{A}\right)^{m-1}\omega\left[\frac{g'h''dv_3 - h'g''dv_2}{(g')^2}\right]$$

so that $$\frac{1}{q_1}\frac{dq_1}{d\omega} = \frac{1}{\omega} + (m - 1)\frac{A}{B}\frac{d(B/A)}{d\omega} + \left(\frac{-1}{\omega + v_3} + \frac{1}{\omega + v_2}\right).$$

From $\omega = \frac{B}{A}\left(\frac{f - v_1 f'}{f'}\right)$ and with $\sigma_f \neq 1$ we have

$$\frac{A}{B}\frac{d(B/A)}{d\omega} = \frac{\sigma_f}{1 - \sigma_f}\left\{\frac{1}{\omega} - \frac{1}{\sigma_f q_1}\frac{dq_1}{d\omega}\right\}.$$

Substituting for $\frac{1}{q_1}\frac{dq_1}{d\omega}$, we obtain

$$\omega\frac{A}{B}\frac{d(B/A)}{d\omega} = \frac{1}{\sigma_f - m}\left\{(1 - \sigma_f) + \frac{\omega(v_3 - v_2)}{(\omega + v_2)(\omega + v_3)}\right\}.$$

Clearly the cases $m = 1$ ($\alpha(A) = C_1A$, $\gamma(B) = C_2B$) and . $= 0$ ($\alpha(A) = C_1$, $\gamma(B) = C_2$) are covered by the above formula. he sign of $\frac{d(B/A)}{d\omega}$ clearly depends crucially on m.

We also note that

$$\frac{\omega}{q_1}\frac{dq_1}{d\omega} = 1 + \frac{(m - 1)}{\sigma_f - m}(1 - \sigma_f) + \frac{\omega(v_3 - v_2)}{(\omega + v_3)(\omega + v_2)}\left[\frac{\sigma_f - 1}{\sigma_f - m}\right]$$

For $m = 0$,

$$\frac{\omega}{q_1}\frac{dq_1}{d\omega} = 1 + \frac{\sigma_f - 1}{\sigma_f}\left[1 + \frac{\omega(v_3 - v_2)}{(\omega + v_3)(\omega + v_2)}\right]$$

Thus for that special case at least, the response of the capital-ıbor ratio in producing the final product to changes in ω depends on the lasticity of substitution and on the relative capital intensities in the ınovating sector.

There are no doubt other "quasi-steady state" situations where omparative dynamic results could be obtained. For example, in the obb-Douglas case, q_1 is determined by ω and is constant over time, but

A/B generally is time dependent in that case. What we have tried to do is to indicate the fact that in the context of a profit-maximizing model, the links between the wage-rental ratio and the direction of innovative activity are generally quite complex and that there are no obvious simple generalizations even under "quasi-steady state" conditions.

FOOTNOTES

1. Nordhaus (1969) has argued convincingly that under the assumptions we make about the manner in which innovations are produced and used, the production of innovations is only consistent with a competitive market structure under unrealistic conditions which also complicate the analysis to an unmanageable degree. On the other hand, the model we employ can be viewed as one in which an independent producer of innovations sells his innovations to a competitive industry, capturing the monopoly profits from the production of innovations. Excluding the case of "big" innovations (see Arrow (1962)), the analysis is identical in either formulation of the model.

2. Note that in this model the firm is <u>certain</u> about all present and future prices, and about the nature of the innovation which will result from the use of specific quantities of inputs to innovative activity.

3. Two important contributions to the theory of induced innovation by Ahmad (1966) and Nordhaus (1973) are left out of this survey because they concentrate on the classification of bias in exogenous trends.

4. We have ignored non-negativity constraints on the control and state variables because of the special assumptions to be imposed on P, F, ϕ, and Ψ as noted below.

5. The first order conditions as stated differ in two crucial respects from those obtained by maximizing the instantaneous rate of cost reduction. First, the explicit consideration of the factor inputs to innovation implies that simply considering what happens to the price of a factor used in final goods production does not provide sufficient information to allow conclusions about bias in technical progress. If that factor is also used in innovation, then additional conditions are needed. Second, the conditions for profit maximization over time depend on more than relative shares.

6. We have assumed a kind of "strong independence" with respect to the production of innovations in that ϕ is independent of B, and Ψ is independent of A. This amounts to assuming the lack of interdependence in research activities with respect to capital- and labor-augmenting innovations, and is at best a simpliflication that is hard to justify except in terms of the ease of manipulation of the model. The same is true of the separability assumptions relating to α and γ.

7. When prices are constant over time and $\dot{q}_1 = 0$, both factors are always augmented at the same proportional rate, independently of the size of ω. When ω is different between time paths, only the relative levels of augmentation, not the rates, are changed. This would appear to support and clarify Fellner's argument.

8. Ahmad (1966) has pointed out the contrast between the results obtained when the IPF relates $\dot{A}/A$ to $\dot{B}/B$ and the results when it relates $\dot{A}$ to $\dot{B}$, and the importance of specifying how the current innovation possibilities depend on past choices (which imply current <u>levels</u> of A and B). Nordhaus (1973) disposes of the relevance of Ahmad's conclusions to growth theory by showing that a necessary condition for balanced growth equilibrium is that the IPF $a = h(b, A, B)$ be independent of B in the sense $\partial h/\partial B = 0$. There is at least a

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surface similarity between the Nordhaus condition and the cases to be examined here. However, it should be emphasized that our model is explicitly concerned with the endogenous production of innovations, while the Ahmad approach is one in which movements of the IPF are determined exogenously, subject only to their dependence on the levels of A and B already achieved. It might be that there is some closer connection between Nordhaus's notion of independence and that employed here, but it has not been possible as yet to discover such a connection.

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